

AD A 102 69 7 Le Catastrophe by any other Ralph V. Buck, Capt., USN

D. STATEURION STATEMENT A

Approved for public release;
Distribution Unlimited



CENTER FOR NAVAL ANALYSES

077270

81 8 11 003

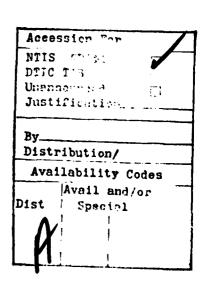


Le Catastrophe by any other name. . . .

Ralph V. Buck, Capt., USN



2000 North Beauregard Street, Alexandria, Virginia 22311



Le Catastrophe by any other name....

R. V. BUCK

The application of the techniques of mathematical physics to the analysis of <u>discontinuities</u> is well known, but it rests on a detailed understanding of the continuous behavior of other parts of the process or system. In many cases, the "soft" sciences especially, such detailed knowledge is unlikely.

As a part of mathematics, Catastrophe Theory (a subset of singularity and bifurcation theories) deals with the properties of discontinuities without reference to any specific underlying mechanism. It is thus most appropriate for the study of systems whose inner workings are not known, and for situations in which the only reliable observations are of the discontinuities (or sudden changes in the state of the system).

René Thom, creator of CT, has admitted that it does not provide a precise quantitative description of what is taking place when a discontinuity/catastrophe occurs, but it does offer a rigorous analytical technique - by classification and analogy - for providing a qualitative interpretation. The instrument sometimes may not fit, or may give misleading interpretations, but it is one of only a very few ways of handling complex systems which tend to "settle" into something like minimum or maximum potential energy states. A major difficulty is in not always knowing whether there is an underlying potential function.

In applying CT, we are limited in the amount of model tinkering we can do, since so few assumptions are made about the system being studied. If we reach unsatisfactory conclusions, we can 1) reject the theory, 2) try to more completely observe and describe the system mechanisms, or 3) change our choice of control variables. Sometimes, as we fit the method to the problem, great insight follows a fortuitous choice of variables.

Borrowing from morphologists, who concern themselves with shapes adopted by natural objects, Thom considered what sorts of surfaces one would observe when there were only one or two behavior variables and no more than four control variables for potential systems posessed of discontinuities. His theory of elementary catastrophes tells us several remarkable things:

- The number of qualitatively different configurations of discontinuities depends not on a potentially large number of state variables but on a generally small number of control variables.
- If the number of control variables is no more than four then there are only seven distinct types of catastrophes.

The seven elementary catastrophes have been described as follows:

Control Factors	Behavior one axis	Variables two axis
1	Fold	-
2	Cusp	-
3	Swallowtail	Hyperbolic Umbilic Elliptic Umbilic
4	Butterfly	Parabolic Umbilic

Since the <u>topological</u> approach provides no scale, an act of faith is required to identify a mathematical jump between stable potential states on a catastrophe behavior surface with an observed discontinuity in nature. In some cases where more is known of the underlying process, some success has been made in developing statistical techniques for fitting CT surfaces to experimental data. Generally, only two assumptions are necessary to suggest a CT application:

- System behavior depends on a limited number of important control factors,
- The system is governed by a potential such that the system is structurally stable except at isolated points.

The elementary catastrophes have been found to model - in the sense of organizing existing observation and suggesting where to look for interesting behavior - a wide variety of processes. One might compare this result to that discovered by the ancient Greeks: only three regular polygons (square, triangle, and hexagon) can be packed edge to edge to fill the plane. This is so, not because geometry dictates to nature, but because there is no other way for the natural process to turn out.

Four quite good introductory readings for those interested in pursuing the geometry, mathematics and applications of CT are:

Woodcock and Davis, Catastrophe Theory, E. P. Dutton, 1978

- P. T. Saunders, An Introduction to Catastrophe Theory, Cambridge Press, 1980.
- E. C. Zeeman, "Catastrophe Theory", Scientific American, April 1976, p.65-82

Behavioral Science, September 1978 issue (applications of Catastrophe Theory in the Behavioral and Life Sciences)

A Combat Example

The combat "laws" of F.W. Lanchester use differential equations and postulate continuous attrition. A difficulty arises, however, in defining when a battle has ended. One way out is to assume a "fight to the finish" rule, though actual combat results suggest there are more generally points at which the "weaker" force breaks off fighting (and perhaps runs away). Many reasons are advanced for such behavior: leadership, training, morale, casualty level, mission, terrain, and C³.

Let us consider some combination of factors which, together, produce unit <u>cohesiveness</u> (C). Let C, along with <u>strength</u> (S) of the unit (inversely proportional to casualties), be considered important control variables which influence combat behavior along a spectrum from "fight" to "run". The behavior variable is called <u>break point</u> (B).

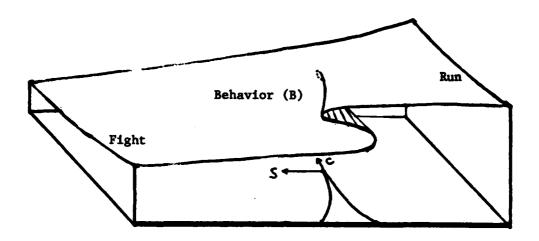
We might then postulate a potential liklihood function which seems to fit a wide variety of combat situations:

Strength	Cohesiveness	Probability
high	normal	
1ow	normal	
high	high	
high/low	low	
		Run Fight (high B) Behavior (low B)

We are perhaps led to consider CT by observing that there are localized discontinuities of behavior in combat, that past history influences whether panic or stiff resistance result from a given force, that sudden flight does not occur at the same values of S and C as sudden resistance, that a slight difference in strength can stimulate a major change in behavior, and that low cohesiveness tends to produce extremes in behavior. All the elements of a cusp catastrophe are now present. We can write the behavior surface equation (origin directly above the cusp vertex) as:

$$B^3 - S - (C \cdot B) = 0$$

with C acting as a splitting factor (bimodal behavior resulting) and S acting as a normal factor (continuous change in behavior results).



One might next proceed to consider tradeoffs between the quality and the quantity of a force, the transient effect of casualties and/or reinforcements during a battle, whether observed data supports collapsing several factors into a "cohesiveness" variable, and what the effect would be of assuming 3 control variables (i.e. casualties, weather, morale) and 2 behavior variables (i.e. perceived ferocity and coherent plan of battle).

One might also speculate whether, for a given estimate of enemy behavior, firepower or propaganda would be the more effective to defeat him.

CHA PROFESSIONAL PAPERS - 1978 TO PRESENT®

PP 211

Mizrahl, Maurice M., "On Approximating the Circular Coverage Function," 14 pp., Feb 1978, AD A054 429

PP 212

Mangel, Marc, "On Singular Characteristic Initial Value Problems with Unique Solution," 20 pp., Jun 1978, AD A058 535

PP 213

Mangel, Marc, "Fluctuations in Systems with Multiple Steady States. Application to Lanchester Equations," 12 pp., Feb 78 (Presented at the First Annual Morkshop on the Information Linkage Between Applied Mathematics and Industry, Naval PG School, Feb 23-25, 1978), AD A071 472

PP 214

steinland, Robert G., "A Somewhat Different View of The Optimal Naval Posture," 37 pp., Jun 1978 (Presented at the 1976 Convention of the American Political Science Association (APSA/IUS Panel on "Changing Strategic Requirements and Military Posture") Chicago, III., September 2, 1976), AD A036 228

PP 215

Colle, Russell C., "Comments on: Principles of information Retrieval by Manfred Kochen," 10 pp., Mar 78 (Published as a Letter to the Editor, Journal of Documentation, Vol. 31, No. 4, pages 298-301), December 1975), AD A

PP 216

Colle, Russell C., "Lotke's Frequency Distribution of Scientitic Productivity," 18 pp., Feb 1978 (Published in the Journal of the American Society for Information Science, Vol. 28, No. 6, pp. 366–370, November 1977), AD A054 425

PP 21

Colle, Russell C., "Bibliometric Studies of Scientific Productivity," 17 pp., Mar 78 (Presented at the Annual meeting of the American Society for Information Science held In Sen Francisco, California, October 1976), AD A

PP 218 - Classified

PP 219

Huntzinger, R. Laver, "Nerket Analysis with Retional Expectetions: Theory and Estimation," 60 pp., Apr 78, AD A054 422

PP 220

Mourer, Doneld E., "Diagonalization by Group Hatrices," $26\ pp.,\ Apr. 78,\ AD\ A054\ 443$

PP 22

Weinland, Robert G., "Superpower Nevel Diplomacy in the October 1973 Arab-israeli Ner," 76 pp., Jun 1978 (Published in Seapower in the Mediterranean: Political Utility and Militery Constraints, The Meshington Papers No. 61, Beverly Hills and London: Sege Publications, 1979) AD A035 364 PP 22

Migrahi, Maurice M., "Correspondence Rules and Peth Integrals," $30~pp_+$, Jun~1978~(Invited paper presented at the CNRS meeting on "Nathematical Problems in Feynman's Path Integrals," Marsellle, France, May 22–26, 1978) (Published In Springer Verlag Lecture Notes in Physics, 106, (1979), 234–233) AD A093-536

PP 22

Hangsi, Marc, "Stochestic Mechanics of Moleculeion Molecule Reactions," 21 pp., Jun 1978, AD A056 227

PP 224

Manger, Marc, "Aggregation, Biturcation, and Extinction in Exploited Animal Populations"," 48 pp., Mar 1978, AD ADS 536
**Portions of this work were started at the institute of

*Portions of this work were started at the institute of Applied Nathematics and Statistics, University of British Columbia, Yencouver, B.C., Canada

PP 225

Mangel, Merc, "Oscillations, Fluctuations, and the Hopf Bifurcation"," 43 pp., Jun 1978, AD A058 537 "Portions of this work were completed at the institute of Applied Mathematics and Statistics, University of British Columbia, Vancouver, Canada.

PP 226

Raiston, J. M. and J. W. Mann, "Temperature and Current Dependence of Degradation in Red-Emitting GeP LEDs," 34 pp., Jun 1978 (Published in Journal of Applied Physics, 50, 3630, May 1979) AD A058 538 "Bail Telephone Laboratories. inc.

P 227

Hangel, Marc, "Uniform Treetment of Fluctuations at Critical Points," 50 pp., May 1978, AD A058 539

PP 228

Mengel, Merc, "Relaxetion at Critical Points: Deterministic and Stochastic Theory," 54 pp., Jun 1978, AD A058 540

PP 22

Mangel, Merc, "Diffusion Theory of Reaction Rates, i: Formulation and Einstein-Smoluchowski Approximation," 50 pp., Jan 1978, AD A058 541

PP 230

Mangel, Marc, "Diffusion Theory of Reaction Rates, il Ornstein-Uhlenbeck Approximation," 34 pp., Feb 1978, AD A058 542

PP 231

Wilson, Desmond P., Jr., "Nevel Projection Forces: The Case for a Responsive MAF," Aug 1978, AD A054 543

PP 232

Jecobson, Louis, "Can Policy Changes & Made Acceptable to Lebor?" Aug 1978 (Submitted for publication in industrial and Labor Relations Review), AD AD61 928

*CMA Professional Papers with an AD number may be obtained from the National Technical Information Service, U.S. Department of Commerce, Springfield, Virginia 22151. Other papers are available from the Management Information Office, Center for Navai Analyses, 2000 North Beauregard Street, Alexandria, Virginia 22311. An Index of Selected Publications is also evaluable on request. The Index Includes a Listing of Professional Papers; with abstracts; Issued from 1969 to June 1981.

PP 233

Jacobson, Louis, "An Alternative Explanation of the Cyclical Pattern of Quits," 23 pp., Sep 1978

PP 234 - Revised

Jondrow, James and Levy, Robert A., "Does Federal Expenditure Displace State and Local Expenditure: The Case of Construction Grants," 25 pp., Oct 1979, AD AD61 529

PP 235

Mizrehl, Maurice M., "The Semiclassical Expansion of the Anharmonic-Oscillator Propagator," 41 pp., Oct 1978 (Published in Journal of Mathematical Physics 20 (1979) pp. 844-855), AD A061 538

PP 237

Meurer, Doneld, "A Matrix Criterion for Normal Integral Bases," 10 pp., Jan 1979 (Published in the Illinois Journal of Mathematics, Vol. 22 (1978), pp. 672-681

PD 234

Utgoff, Kathleen Classen, "Unemployment Insurance and The Employment Rate," 20 pp., Oct 1978 (Presented at the Conference on Economic Indicators and Performance: The Current Dilemme Facing Government and Business Leaders, presented by Indiana University Graduate School of Business). AD AD61 527

PP 230

Trost, R. P. and Merner, J. T., "The Effects of Military Occupational Training on Civilian Earnings: An income Selectivity Approach," 38 pp., Nov 1979k, AD A077 831

DD 240

Powers, Bruce, "Goals of the Center for Neval Analyses," 13 pp., Dec 1978, AD A063 759

PP 241

Mangel, Marc, "Fluctuations at Chemical Instabilities," 24 pp., Dec 1978 (Published in Journal of Chemical Physics, Vol. 69, No. 8, Oct 15, 1978). AD A063 787

PP 242

Simpson, William R., "The Analysis of Dynamically Interactive Systems (Air Combat by the Numbers)," 160 pp., Dec 1978. AO AO63 760

PP 243

Simpson, William R., "A Probabilistic Formulation of Murphy Dynamics as Applied to the Analysis of Operational Research Problems," 18 pp., Dec 1978, AD A063 761

PP 244

Shermen, Allen and Horoxitz, Stanley A., "Meintenance Costs of Complex Equipment," 20 pp., Dec 1978 (Published By The American Society of Nevel Engineers, Nevel Engineers Journel, Vol. 91, No. 6, Dec 1979) AD AO71 473

PP 245

Simpson, Milliam R., "The Accelerometer Methods of Obtaining Aircraft Performance from Filight Test Date (Dynamic Performance Testing)," 403 pp., Jun 1979, AD A075 226

PP 246

Brechiling, Frank, "Layoffs and Unemployment Insurance," 35 pp., Feb 1979 (Presented at the Noer Conference on "Low Income Labor Markets," Chicago, Jun 1978), AD AD96 629

PP 246

Thomas, James A., Jr., "The Transport Properties of Dilute Games in Applied Fields," 183 pp., Nor 1979, AD A096 464 PP 249

Glesser, Kenneth S., "A Secretary Problem with a Rendom Number of Cholces," 23 pp., Mar 1979

PP 250

Mangel, Marc, "Modeling Fluctuations in Macroscopic Systems," $26~\rho p$., Jun 1979

PP 25

Trost, Robert P., "The Estimation and interpretation of Several Selectivity Models," 37 pp., Jun 1979, AD A075 941

PP 252

Nunn, Walter R., "Position Finding with Prior Knowledge of Coverlence Parameters," 5 pp., Jun 1979 (Published in IEEE Transactions on Aerospace & Electronic Systems, Vol. AES-15, No. 3. Mar 1979

PP 29

Glasser, Kenneth S., "The d-Choice Secretary Problem," 32 pp., Jun 1979, AD A075 225

PP 254

Mangel, Marc and Quanback, David B., "integration of a Bivariate Normal Over an Offset Circle," 14 pp., Jun 1979, AD A096 471

- PP 255 Classified, AD 8051 441L
- PP 256

Maurer, Donald E., "Using Personnel Distribution Models," 27 pp., Feb 1980, AD A062 218

PP 257

Theier, R., "Discounting and Fiscal Constraints: Why Discounting is Always Right," 10 pp., Aug 1979, AD A075 224

PP 258

Mangel, Marc S. and Thomas, James A., Jr., "Analytical Methods in Search Theory," 86 pp., Nov 1979, AD A077 832

DD 25

Glass, David V.; Hsu, ih-Ching; Nunn, Neiter R., and Perin, David A., "A Class of Commutative Markov Hatrices," 17 pp., Nov 1979, AD AD77 833

PP 26

Mangel, Marc S. and Cope, Devis K., "Detection Rate and Sweep Width in Visual Search," 14 pp., Nov 1979, AD A077 834

PP 26

Vile, Carlos L.; Zvijec, David J. and Ross, John, "Franck-Condon Theory of Chemical Dynamics. Vi. Angular Distributions of Reaction Products," 14 pp., Nov 1979 (Reprinted from Journal Chemical Phys. 70(12), 15 Jun 1979), AD AD78-287

PP 26

Petersen, Cherles C., "Third World Military Elites in Soviet Perspective," 50 pp., Nov 1979, AD A077 835

PP 26

Robinson, Kethy I., "Using Commercial Tankers and Containerships for Navy Underway Replenishment," 25 pp., Nov 1979, AD A077 836

Weinland, Robert G., "The U.S. Nevy in the Pacific: Past, Present, and Gilmpses of the Future," 31 pp., Nov 1979 (Delivered at the International Symposium on the Sea, sponsored by the International Institute for Strategic Studies, The Brockings Institution and the Yomiuri Shimbun, Takyo, 16-20 Oct 1978) AD A066 837

Weinland, Robert G., "Mar and Peace in the North: Some Political Implications of the Changing Military Situation in Northern Europe," 18 pp., Nov 1979 (Prepared for presentation to the Conference of the Nordic Balance in The Changing Military and Political Perspective: Situation," Center for Strategic and International Studies, Georgetown University, Jun 15-16, 19781 AD A077 838

Utgoff, Kathy Classen, and Brechling, Frank, "Taxes and Inflation," 25 pp., Nov 1979, AD A081 194

Trost, Robert P., and Vogel, Robert C., "The Response of State Government Receipts to Economic Fluctuations and the Allocation of Counter-Cyclical Revenue Sharing Grants,* 12 pp+, Dec 1979 (Reprinted from the Review of Economics and Statistics, Vol. LX1, No. 3, August 1979)

Thomason, James S., "Seaport Dependence and Inter-State Cooperation: The Case of Sub-Saharan Africa," 141 pp., Jan 1980, AD A081 193

PP 269

Weiss, Kenneth G., "The Soviet Involvement in the Ogaden War," 42 pp., Jan 1980 (Presented at the Southern Conference on Slavic Studies in October, 1979), AD A082 219

Remnek, Richard, "Soviet Policy in the Horn of Africa: The Decision to intervene, $^{\rm H}$ 52 pp., Jan 1980 (To be published in "The Soviet Union in the Third World: Success or Failure," ed. by Robert H. Donaldson, Westview Press, Boulder, Co., Summer 1980), AD A081 195

McConnell, James, "Soviet and American Strategic Doctrines: One More Time," 43 pp., Jan 1980, AD A081 192

Weiss, Kenneth G., "The Azores in Diplomecy and Strategy, 1940-1945, 46 pp., Mar 1980, AD A085 094

Makeda, Michael K., "Lebor Supply of Wives with Hysbands Employed Either Full Time or Part Time," 39 pp., Nor 1980, AD A082 220

Goldberg, Laurence, "Recruiters Advertising and Nevy Enlistments," $34~\rm pp$., Mar 1980, AD A082 221

Coldbarg, Lawrence, "Delaying an Overheut and Ship's Equipment," 40 pps, May 1980, AD A085 095

Mangel, Marc, "Small Fluctuations in Systems with Multiple Limit Cycles," 19 pp., Mar 1980 (Published in SIAM J. Appl. Maths, Vol. 38, No. 1, Feb 1980) AD A086 229

Mizrehl, Meurice, "A Targeting Problem: Exact vs. Expected-Value Approaches," 23 pp., Apr 1980, AD A085 096

Walt, Stephen M., "Causal Inferences and the Use of Force: A Criffque of Force Without Wer," 50 pp., May 1980, AD A085 097

PP 280

Goldberg, Lawrence, "Estimation of the Effects of A Ship's Steaming on the Fallure Rate of its Equipment: An Application of Econometric Analysis," 25 pp., Apr 1980, AD A085 098

Mizrahl, Meurice M., "Comment on 'Discretization Problems of Functional Integrals in Phase Space¹, " 2 pp., May 1980, AD A094 994

PP 283

Dismukes, Bredford, "Expected Demand for the U.S. Navy to Serve as An Instrument of U.S. Foreign Policy: Thinking About Political and Military Environmental Factors, * 30 pp., Apr 1980, AD A085 099

PP 284

J. Kelison, * W. Munn, and U. Sumita, ** "The Laguerre Transform," 119 pp., May 1980, AD A085 100

- *The Graduate School of Management, University of Rochester and the Center for Nevel Analyses
- **The Graduate School of Management, University of Rochester

Remnek, Richard B., "Superpower Security Interests in the Indian Ocean Area," 26 pp., Jun 1980, AD A087 113

PP 286

Mizrahi, Maurice M., "On the WKB Approximation to the Propagator for Arbitrary Hamiltonians," 25 pp., Aug 1980 (Published in Journal of Math. Phys., 22(1) Jan 1981), AD A091 307

PP 287

Cope, Devis, "Limit Cycle Solutions of Reaction-Diffusion Equations," 35 pp., Jun 1980, AD A087 114

PP 288

Golman, Welter, "Don't Let Your Siides Filip You: A Painless Guide to Visuals That Really Ald," 28 pp., Oct 1980, AD A092 732

PP 289

Robinson, Jack, "Adequate Classification Guidence - A Solution and a Problem, 7 pp., Aug 1980, AD A091 212

Watson, Gregory H., "Evaluation of Computer Software in an Operational Environment," 17 pp., Aug 1980, AD A091 213

Maddele, G. S.º and Trost, R. P., "Some Extensions of the Nariove Press Model," 17 pp., Oct 1980, AD A091 946 University of Florida

PP 292

Thomes, James A., Jr., "The Transport Properties of Binary Ges Mixtures in Applied Magnetic Fields," 10 pp., Sept 1980 (Published in Journal of Chemical Physics 72(10), 15 May 1980

PP 293

Thomes, James A., Jr., "Evaluation of Kinetic Theory Collision integrals Using the Generalized Phase Shift Approach," 12 pp., Sept 1980 (Printed in Journal of Chemical Physics 72(10), 15 May 1980

PP 29/

Roberts, Stephen S., "French Neval Policy Outside of Europe," 30 pp., Sept 1980 (Presented at the Conference of the Section on Military Studies, International Studies Association Klaweh (stand, S-C-), AD A091 306

PP 29

Roberts, Stephen S., "An Indicator of Informal Empire: Patterns of U.S. Nevy Cruising on Overseas Stations, 1869– 1897," 40 pp., Sept 1980 (Presented at Fourth Naval History Symposium, US Naval Academy, 26 October 1979, AD A091 316

PP 296

Dismukes, Bredford and Petersen, Charles C., "Maritime Factors Affecting Iberian Security," (Factores Maritimos Que Afectan La Securidad Ibelica) 14 pp., Oct 1980, AD A092 733

- PP 297 Classified
- PP 208

Mizrahi, Maurice M., "A Markov Approach to Large Missile Attacks," 31 pp., Jan 1981, AD A096,159

PP 299

Jondrow, James M. and Levy, Robert A., Mwage Leadership in Construction, 19 pp., Jan 1981, AD A094 797

00 10

Jondros, James and Schmidt, Peter,* "On the Estimation of Technical Inefficiency in the Stochestic Frontier Production Function Model," 11 pps., Jan 1981, AD A096 159 "Michigan State University

PP 301

Jondrow, James M.; Levy, Robert A. and Hughes, Ciefre, "Technical Change and Employment in Steel, Autos, Aluminum, and from Ore, 17 pp., Mar 1981, AD A099 394

PP 302

Joudnay, James M. and Levy, Robert A., "The Effect of Imports on Employment Under Retional Expectations," 19 pp., Apr 1981, AD A099 392

PP 304

Duffy, Michael K.; Greenrood, Michael J.* and McDorell, John M., ** ** A Cross-Sectional Model of Annual Interregional Migration and Employment Growth: Intertemporal Evidence of Structural Change, 1938-1975, ** 31 pp., Apr 1981, AD A099 393 **University of Colorado ***Arizona State University

PP 305

 $N_{\rm Unin},\ Leura$ H., "An introduction to the Literature of Search Theory," 32 pp., Jun 1981

PP 306

Anger, Thomes E., "What Good Are Werfere Models?" 7 pp., May 1981

PP 307

Thomason, James, "Dependence, Risk, and Vulnerability," 43 pps, Jun 1961

PP 309

Meinland, Robert G., "An (The?) Explanation of the Soviet invasion of Afghanistan," 44 pp., May 1961

PP 310

Stanford, Jenette M. and Tal To Mu, * "A Predictive Method for Determining Possible Three-dimensional Foldings of Immunoglobulin Beckbones Around Antibody Combining Sites, * 19 pp., Jun 1981 (Published in J. Theor. Biol. (1981) 88, 421-439

*Northwestern University, Evanston, IL

PP 311

Bowes, Martanne, Brechiing, Frank P. R., and Utgoff, Kathleen P. Classen, "An Evaluation of UI Funds," 13 pp., May 1981 (Published in National Commission on Unemployment Compensation's Unemployment Compensation: Studies and Research, Volume 2, July 1980)

PP 312

Jondros, James; Bowes, Marianne and Levy, Robert, "The Optimum Speed Limin," 23 pp., Jun 1981

PP 313

Roberts, Stephen S., "The U.S. Nevy In the 1980s," 36 pp.,

PP 315

Buck, Raiph V., Capt., "Le Cetastrophe by any other name...," Jul 1981